

ROBUST DARK ENERGY CONSTRAINTS FROM SUPERNOVAE, GALAXY CLUSTERING, AND THREE-YEAR WILKINSON MICROWAVE ANISOTROPY PROBE OBSERVATIONS

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ABSTRACT

Type Ia supernova (SN Ia), galaxy clustering, and cosmic microwave background anisotropy (CMB) data provide complementary constraints on the nature of the dark energy in the universe. We find that the three-year Wilkinson Microwave Anisotropy Probe (WMAP) observations give a CMB shift parameter of $R \equiv (\Omega_m H_0^2)^{1/2} \int_0^{z_{CMB}} dz'/H(z') = 1.70 \pm 0.03$. Using this new measured value of the CMB shift parameter, together with the baryon acoustic oscillation (BAO) measurement from the Sloan Digital Sky Survey (SDSS), and SN Ia data from the HST/GOODS program and the first year Supernova Legacy Survey, we derive model-independent constraints on the dark energy density $\rho_X(z)$ and the cosmic expansion rate $H(z)$. We also derive constraints on the dark energy equation of state $w_X(z) = w_0 + w'z$ (with cutoff at $z = 2$) and $w_X(a) = w_0 + (1 - a)w_a$.

We find that current data provide slightly tighter constraints on $\rho_X(z)$ and $H(z)$ as free functions in redshift, and roughly a factor of two improvement in constraining $w_X(z)$. A cosmological constant remains consistent with data, however, uncertainties remain large for model-independent constraints of dark energy. Significant increase in the number of observed SNe Ia between redshifts of 1 and 2, complemented by improved BAO and weak lensing cosmography measurements (as expected from the JEDI mission concept for the Joint Dark Energy Mission), will be required to dramatically tighten model-independent dark energy constraints.

Subject headings: Cosmology

1. INTRODUCTION

The observed accelerated expansion of the universe (Riess et al. 1998; Perlmutter et al. 1999) can be explained by an unknown energy component in the universe (Freese et al. 1987; Linde 1987; Peebles & Ratra 1988; Wetterich 1988; Frieman et al. 1995; Caldwell, Dave & Steinhardt 1998), or a modification of general relativity (Sahni & Habib 1998; Parker & Raval 1999; Dvali, Gabadadze, & Porrati 2000; Mersini, Bastero-Gil, & Kanti 2001; Freese & Lewis 2002). Padmanabhan (2003) and Peebles & Ratra (2003) contain reviews of many models. Some recent examples of models are presented in Carroll et al. (2004); Onemli & Woodard (2004); Cardone et al. (2005); Kolb, Matarrese, & Riotto (2005); Martineau & Brandenberger (2005); McInnes (2005); Cai, Gong, & Wang (2006). For convenience, we refer to the cause for the cosmic acceleration as “dark energy”.

Data of Type Ia supernovae (SNe Ia), cosmic large scale structure (LSS), and the cosmic microwave anisotropy (CMB) are complementary in precision cosmology (Bahcall et al. 1999; Eisenstein, Hu, & Tegmark 1999; Wang, Spergel, & Strauss 1999). An important development in this complementarity is to use the baryonic acoustic oscillations (BAO) in the galaxy power spectrum as a cosmological standard ruler to probe dark energy (Blake & Glazebrook 2003; Seo & Eisenstein 2003).

In placing robust constraints on dark energy, it is important to (1) derive model-independent dark energy constraints (Wang & Garnavich 2001; Tegmark 2002; Daly & Djorgovski 2003), and (2) use data derived *without* assuming dark energy to be a cosmological constant (Wang &

Tegmark 2004).

In this paper, we derive the CMB shift parameter from the three-year Wilkinson Microwave Anisotropy Probe (WMAP) observations (Spergel et al. 2006; Bennett et al. 2003), and show that its measured value is mostly independent of assumptions made about dark energy. Using this new measured value of the CMB shift parameter, together with the LSS data from the BAO measurement from the Sloan Digital Sky Survey (SDSS), and SN Ia data from the HST/GOODS program and the first year Supernova Legacy Survey (SNLS), we derive model-independent constraints on the dark energy density $\rho_X(z)$ and the cosmic expansion rate $H(z)$. For reference and comparison, we also derive constraints on the linear dark energy equation of state $w_X(z) = w_0 + w'z$ (with cutoff at $z = 2$) and $w_X(a) = w_0 + (1 - a)w_a$ (Chevallier & Polarski 2001).

Sec.2 describes the method and data used in our calculations. We present results in Sec.3 and summarize in Sec.4.

2. THE METHOD AND DATA USED

2.1. The method

We run a Monte Carlo Markov Chain (MCMC) based on the MCMC engine of Lewis & Bridle (2002) to obtain $\mathcal{O}(10^6)$ samples for each set of results presented in this paper. The chains are subsequently appropriately thinned.

We derive constraints on the dark energy density $\rho_X(z)$ as a free function, with its value at redshifts z_i , $\rho_X(z_i)$, treated as independent parameters estimated from data. For $z > z_{\text{cut}}$, we assume $\rho_X(z)$ to be a powerlaw smoothly

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matched on to $\rho_X(z)$ at $z = z_{\text{cut}}$ (Wang & Tegmark 2004):

$$\rho_X(z) = \rho_X(z_{\text{cut}}) \left(\frac{1+z}{1+z_{\text{cut}}} \right)^\alpha. \quad (1)$$

The number of observed SNe Ia is either very few or none beyond z_{cut} . For the Riess et al. (2004) sample, $z_{\text{cut}} = 1.4$. For the Astier et al. (2005) sample, $z_{\text{cut}} = 1.01$. We use cubic spline interpolation to obtain values of $\rho_X(z)$ at other values of z (Wang & Tegmark 2004).

The $H(z)$ values corresponding to the estimated $\rho_X(z_i)$ are estimated directly from the MCMC chain to fully incorporate the correlation between the estimated parameters, and compared with the uncorrelated estimates of $H(z)$ from SN Ia data only (Wang & Tegmark 2005).

For reference and comparison with the work by others, we also derive constraints on dark energy models with a constant dark energy equation of state w ($w_X(z) = w$), and a linear equation of state parametrized by (1) $w_X(z) = w_0 + w'z$ at $z \leq 2$, and $w_X(z) = w_0 + 2w'$ at $z > 2$; (2) $w_X(z) = w_0 + (1-a)w_a$.

2.2. SN Ia data

Calibrated SN Ia data (Phillips 1993; Riess, Press, & Kirshner 1995) give luminosity distances $d_L(z_i)$ to the redshifts of the SNe Ia z_i . For a flat universe

$$d_L(z) = cH_0^{-1}(1+z) \int_0^z \frac{dz'}{E(z')}, \quad (2)$$

where

$$E(z) \equiv [\Omega_m(1+z)^3 + (1-\Omega_m)\rho_X(z)/\rho_X(0)]^{1/2}, \quad (3)$$

with $\rho_X(z)$ denoting the dark energy density.

We use SN Ia data from the HST/GOODS program (Riess et al. 2004) and the first year SNLS (Astier et al. 2005), together with nearby SN Ia data. The comparison of results from these two data sets provides a consistency check.

We do not combine these two SN Ia data sets, as they have systematic differences in data processing. Fig.1 shows the difference in estimated distance moduli for 37 SNe Ia included in both the Riess et al. (2004) “gold” sample and the Astier et al. (2005) sample; there is clearly significant scatter due to difference in analysis techniques (Wang 2000b). As a result, the two data sets have noticeably different zero point calibrations. A given zero point calibration affects the measurement of H_0 , but has no impact on dark energy constraints (see Eq.[2]). Combining the data sets will lead to artificial systematic errors resulting from the difference in zero point calibrations, which are difficult to quantify; this outweighs the gain in accuracy at present since there is a large overlap between the two data sets. It is important to use data analyzed using the same technique (which corresponds to the same zero point calibration); although it would be useful to use the same data analyzed using one or more other techniques (but only one technique should be used at a time) for cross check.

We use the Riess et al. (2004) “gold” sample flux-averaged with $\Delta z = 0.05$. This sample includes 9 SNe

Ia at $z > 1$, and appears to have systematic effects that would bias the distance estimates somewhat without flux-averaging (Wang & Tegmark 2004; Wang 2005). Flux-averaging (Wang 2000b; Wang & Mukherjee 2004) removes the bias due to weak lensing magnification of SNe Ia (Kantowski, Vaughan, & Branch 1995; Frieman 1997; Wambsganss et al. 1997; Holz 1998; Metcalf & Silk 1999; Wang 1999; Barber et al. 2000; Vale & White 2003)³, or other systematic effects that mimics weak lensing qualitatively in affecting the observed SN Ia brightness.

We have added a conservative estimate of 0.15 mag in intrinsic dispersion of SN Ia peak brightness in quadrature to the distance moduli published by Astier et al. (2005), instead of using the smaller intrinsic dispersion derived by Astier et al. (2005) by requiring a reduced $\chi^2 = 1$ in their model fitting. This is because the intrinsic dispersion in SN Ia peak brightness should be derived from the distribution of nearby SNe Ia, or SNe Ia from the same small redshift interval if the distribution in SN Ia peak brightness evolves with cosmic time. This distribution is not well known at present, but will become better known as more SNe Ia are observed by the nearby SN Ia factory (Aldering et al. 2002) and the SNLS (Astier et al. 2005). By using the larger intrinsic dispersion, we allow some margin for the uncertainties in SN Ia peak brightness distribution.

2.3. LSS data

For LSS data, we use the measurement of the BAO peak in the distribution of SDSS luminous red galaxies (LRG’s). We do not use the linear growth rate measured by the 2dF survey, as there are some inconsistencies in currently published 2dF results (Verde et al. 2002; Hawkins et al. 2003).⁴

The SDSS BAO measurement (Eisenstein et al. 2005) gives $A = 0.469(n_S/0.98)^{-0.35} \pm 0.017$ (independent of a dark energy model) at $z_{\text{BAO}} = 0.35$, where A is defined as

$$A = \left[r^2(z_{\text{BAO}}) \frac{cz_{\text{BAO}}}{H(z_{\text{BAO}})} \right]^{1/3} \frac{(\Omega_m H_0^2)^{1/2}}{cz_{\text{BAO}}}, \quad (4)$$

where $r(z)$ is the comoving distance, and $H(z)$ is the Hubble parameter. Note that $H(z) = H_0 E(z)$. We take the scalar spectral index $n_S = 0.95$ as measured by WMAP3 (Spergel et al. 2006). Note that this constraint from Eisenstein et al. (2005) is not just a simple measurement of the BAO feature; it also relies on the constraints on $\Omega_m h^2$ from measuring the power spectrum turnover scale (related to matter-radiation equality). The latter makes their BAO constraint less robust than it would be otherwise. A new analysis of the SDSS data to derive truly robust BAO constraints would be very useful for placing dark energy constraints (Dick, Knox, & Chu 2006).

Also note that the Eisenstein et al. (2005) constraint on A depends on the scalar spectral index n_S . Since the error on n_S from WMAP data does not increase the effective error on A , and the correlation between n_S and the CMB shift parameter R is weak, we have ignored the very weak correlation between A and R in our likelihood analysis.

³ Weak lensing magnification of SNe Ia can also be used as a cosmological probe, see Dodelson & Vallinotto (2005); Cooray, Holz, & Huterer (2006); Munshi & Valageas (2006).

⁴ Combining the results from Verde et al. (2002) and Hawkins et al. (2003), we would obtain the linear growth rate $f(z_{2\text{dF}}) = 0.51 \pm 0.11$. However, Hawkins et al. (2003) points out that the Verde et al. (2002) results strongly depend on the assumed pairwise peculiar velocity dispersion of 385 km/s, while Hawkins et al. (2003) finds the pairwise peculiar velocity dispersion to be 500 km/s. A more self-consistent linear growth rate from 2dF data has to await a new bispectrum analysis of the 2dF data.

We have derived R from WMAP data marginalized over all relevant parameters.

2.4. CMB data

The CMB shift parameter R is perhaps the least model-dependent parameter that can be extracted from CMB data, since it is independent of H_0 . The shift parameter R is given by (Bond, Efstathiou, & Tegmark 1997)

$$R \equiv \Omega_m^{1/2} \int_0^{z_{CMB}} dz' / E(z'), \quad (5)$$

where z_{CMB} is the redshift of recombination; thus $R = (\Omega_m H_0^2)^{1/2} r(z_{CMB})/c$ (for a flat universe), which is well determined since both $\Omega_m h^2$ and $r(z_{CMB})$ are accurately determined by CMB data. Similar reasoning applies to an open and a closed universe as well.

The ratio of the sound horizon at recombination to $r(z_{CMB})$, θ_s , should be more or less equivalent to R . We call R the “CMB shift parameter” following previously published literature over the last 9 years (see for example, Bond, Efstathiou, & Tegmark (1997); Odman et al. (2003)). One advantage of R is that it only involves a simple integral over $1/E(z)$, while the sound horizon at last scattering is more complicated to calculate accurately and depends on more parameters.

We compute R using the MCMC chains from the analysis of the three year WMAP data provided by the WMAP team (Spergel et al. 2006). The resultant probability distribution of R for three different classes of models are shown in Fig.2. Clearly, the measured R from WMAP 3 year data has only a very weak model dependence⁵.

We use a Gaussian distribution in R with $\langle R \rangle = 1.70$ and $\sigma_R = 0.03$ (thin solid line in Fig.2) in deriving our results presented in Sec.3. This fits the WMAP 3 year data well, and allows some margin in error for the very weak model dependence of R .

3. RESULTS

In deriving all the results presented in this section, we have assumed a flat universe, and marginalized over Ω_m and h .

Fig.3 shows $\rho_X(z)$ measured using SN Ia data (Riess et al. 2004; Astier et al. 2005), combined with the WMAP 3 year data, and the SDSS BAO data. Note that beyond $z_{\text{cut}}=1.4$ (upper panel) and 1.01 (lower panel), $\rho_X(z)$ is parametrized by a power law $(1+z)^\alpha$, the index of which is marginalized over.

Fig.4 shows the uncorrelated $H(z)$ estimated using only SN Ia data (Wang & Tegmark 2005), and the $H(z)$ corresponding to the $\rho_X(z_i)$ shown in Fig.3.

Tables 1 and 2 give the 68% confidence intervals of $\rho_X(z_i)$, α , and $H(z_i)$ for the WMAP 3 year and the SDSS BAO data, combined with SN Ia data from Riess et al. (2004) and Astier et al. (2005) respectively.

Tables 3 and 4 give the covariance matrices for $(\rho_X(z_i), \alpha)$, and $(H(z_i), \alpha)$ for the WMAP3 and SDSS BAO data combined with SN Ia data from Riess et al. (2004) and Astier et al. (2005) respectively.

Table 5 gives the constraints on a constant w ($w_X(z) = \text{const.}$), (w_0, w') ($w_X(z) = w_0 + w'z$ at $z \leq 2$,

and $w_X(z) = w_0 + 2w'$ at $z > 2$), and (w_0, w_a) ($w_X(z) = w_0 + (1-a)w_a$).

Fig.5 shows the 68% and 95% joint confidence contours for (w_0, w') and (w_0, w_a) .

4. DISCUSSION AND SUMMARY

In order to place robust constraints on dark energy in a simple and transparent manner, we have derived the CMB shift parameter from the WMAP 3 year data ($R = 1.70 \pm 0.03$, see Fig.2). We constrain dark energy using this new measurement of the CMB shift parameter, together with LSS data (the BAO measurement from the SDSS LRG's), and SN Ia data (from the HST/GOODS program and the first year SNLS).

We have derived model-independent constraints on the dark energy density $\rho_X(z)$ and the cosmic expansion rate $H(z)$ (Figs.3-4 and Tables 1-4). There are two reasons that one should use $\rho_X(z)$ and $H(z)$ instead of $w_X(z)$ to probe dark energy. First, $\rho_X(z)$ and $H(z)$ are more directly related to observables than $w_X(z)$ (see Eqs.[2]-[5], and note that $\partial \ln \rho_X / \partial \ln a = -3(1 + w_X)$) (Wang & Garnavich 2001; Tegmark 2002). This means that $\rho_X(z)$ is more tightly constrained by data than $w_X(z)$ (Wang & Freese 2006; Daly & Djorgovski 2005; Huterer & Cooray 2005; Ishak 2005; Dick, Knox, & Chu 2006). Secondly, $\rho_X(z)$ and $H(z)$ are more general phenomenological representations of dark energy than $w_X(z)$ (Wang & Tegmark 2004). One must integrate the equation, $\partial \ln \rho_X / \partial \ln a = -3(1 + w_X)$, to obtain $\rho_X(z)$ before a comparison with data can be made, hence even arbitrary function $w_X(z)$ has the hidden assumption that $\rho_X(z)$ is non-negative (since $\rho_X(0) \geq 0$). Since we don't know what dark energy is – it may not even be energy at all but a modification of general relativity – $\rho_X(z)$ may well have been negative at some past epoch or become negative in the future. Measuring $\rho_X(z)$ and $H(z)$ (instead of $w_X(z)$) from data allows us to constrain this possibility.

For comparison with the work by others, we also derive constraints on the dark energy equation of state $w_X(z) = w_0 + w'z$ (with cutoff at $z = 2$) and $w_X(z) = w_0 + (1-a)w_a$ (Fig.5 and Table 5).

Because of the difference in SN Ia analysis techniques used in deriving the Riess et al. (2004) “gold” sample and Astier et al. (2005) sample (see Fig.1), we have presented the results for these two data sets separately. This is necessary in order to avoid introducing systematic errors due to the different zero point calibrations of the two data sets (which are difficult to quantify) (Wang 2000b). We find that the Riess et al. (2004) and Astier et al. (2005) data sets give similar and consistent constraints on dark energy (Figs.3-5), with the Riess et al. (2004) sample being able to constrain $\rho_X(z)$ and $H(z)$ at $z > 1$ since it contains 9 SNe Ia at $z > 1$. This is reassuring, and seems to indicate that the Riess et al. (2004) sample (after flux-averaging) is consistent with the Astier et al. (2005) sample. It will be useful to derive the distances to *all* the SNe Ia using the *same* technique to reduce systematic uncertainties (Wang 2000b) and maximize the number of SNe Ia that can be used in the same dark energy analysis.

⁵ For a Bayesian analysis of the number of parameters required by current cosmological data, in other words of what comprises an adequate model, see Mukherjee, Parkinson, & Liddle (2006)

TABLE 1

THE MEAN AND THE 68% CONFIDENCE INTERVALS OF $\rho_X(z_i)$, α , AND $H(z_i)$ FOR THE CMB AND LSS DATA COMBINED WITH SN IA DATA FROM RIESS ET AL. (2004).

Parameter	Riess04+WMAP3+SDSS
$\rho_X(0.467)$	1.159 (0.953, 1.361)
$\rho_X(0.933)$	1.357 (0.602, 2.095)
$\rho_X(1.400)$	2.751 (0.408, 5.053)
α	-0.037 (-1.927, 1.818)
$H(0.467)$	1.321 (1.257, 1.384)
$H(0.933)$	1.758 (1.617, 1.896)
$H(1.400)$	2.419 (2.045, 2.797)

TABLE 2

THE MEAN AND THE 68% CONFIDENCE INTERVALS OF $\rho_X(z_i)$, α , AND $H(z_i)$ FOR THE CMB AND LSS DATA COMBINED WITH SN IA DATA FROM ASTIER ET AL. (2005).

Parameter	Astier05+WMAP3+SDSS
$\rho_X(0.505)$	1.013 (0.893, 1.131)
$\rho_X(1.010)$	1.579 (0.833, 2.327)
α	0.030 (-1.786, 1.770)
$H(0.505)$	1.290 (1.252, 1.327)
$H(1.010)$	1.828 (1.678, 1.978)

TABLE 3

THE COVARIANCE MATRICES FOR $\rho_X(z_i)$ AND $H(z_i)$ FROM THE WMAP 3 YEAR AND THE SDSS BAO DATA COMBINED WITH SN IA DATA FROM RIESS ET AL. (2004).

	$\rho_X(0.467)$	$\rho_X(0.933)$	$\rho_X(1.400)$	α
$\rho_X(0.467)$	0.426E-01	0.226E-01	-0.137E+00	0.153E-01
$\rho_X(0.933)$	0.226E-01	0.585E+00	-0.261E+00	-0.322E+00
$\rho_X(1.400)$	-0.137E+00	-0.261E+00	0.717E+01	-0.110E+01
α	0.153E-01	-0.322E+00	-0.110E+01	0.276E+01
	$H(0.467)$	$H(0.933)$	$H(1.400)$	α
$H(0.467)$	0.407E-02	0.185E-02	0.121E-02	0.851E-02
$H(0.933)$	0.185E-02	0.200E-01	-0.130E-02	-0.534E-01
$H(1.400)$	0.121E-02	-0.130E-02	0.154E+00	-0.125E+00
α	0.851E-02	-0.534E-01	-0.125E+00	0.276E+01

We find that compared to previous results in Wang & Tegmark (2004), the current data provide a slightly tighter constraint on $\rho_X(z)$ and $H(z)$ as free functions of cosmic time, and roughly a factor of two improvement in constraining $w_X(z)$. Note that the Astier et al. (2005) data set (together with WMAP 3 year and SDSS BAO data) provides a tighter constraint on $w_0 \equiv w_X(z=0)$ because it contains more SNe Ia at lower redshifts. Because of the strong correlation between w' (or w_a) and w_0 , this has led to tighter constraints on w' and w_a as well.

A cosmological constant remains consistent with data, although more exotic possibilities are still allowed, consistent with previous results (see for example, Wang & Tegmark (2004, 2005); Alam & Sahni (2005); Daly

& Djorgovski (2005); Jassal, Bagla, & Padmanabhan (2005a,b); Dick, Knox, & Chu (2006); Ichikawa & Takahashi (2006); Jassal, Bagla, & Padmanabhan (2006); Nesseris & Perivolaropoulos (2006); Schimd et al. (2006); Wilson, Chen, & Ratra (2006)). In particular, Zhao et al. (2006) uses WMAP 3 year, SN Ia data from Riess et al. (2004), together with SDSS 3D power spectra and Lyman- α forest information data, and found constraints on $w_X(z) = w_0 + w_1 z / (1+z)$ that are qualitatively consistent with our results, with significant differences that can be explained by the differences in the combination of data used, and perhaps some data analysis details.

We find that uncertainties remain large for model-independent constraints of dark energy (see Figs.3-4). Sig-

TABLE 4

THE COVARIANCE MATRICES FOR $\rho_X(z_i)$ AND $H(z_i)$ FROM THE WMAP 3 YEAR AND THE SDSS BAO DATA COMBINED WITH SN IA DATA FROM ASTIER ET AL. (2005).

	$\rho_X(0.505)$	$\rho_X(1.010)$	α
$\rho_X(0.505)$	0.148E-01	0.472E-01	-0.750E-01
$\rho_X(1.010)$	0.472E-01	0.559E+00	-0.460E+00
α	-0.750E-01	-0.460E+00	0.254E+01
	$H(0.505)$	$H(1.010)$	α
$H(0.505)$	0.146E-02	0.308E-02	-0.118E-01
$H(1.010)$	0.308E-02	0.223E-01	-0.729E-01
α	-0.118E-01	-0.729E-01	0.254E+01

TABLE 5

THE MEAN AND 68% AND 95% CONFIDENCE LEVEL CONSTRAINTS ON A CONSTANT w AND (w_0, w') , AND THE COVARIANCE BETWEEN w_0 AND w' , AND BETWEEN w_0 AND w_a .

	Riess04+WMAP3+SDSS	Astier05+WMAP3+SDSS
w	-0.885 ^{+0.109 +0.206} _{-0.111 -0.227}	-0.999 ^{+0.082 +0.159} _{-0.083 -0.168}
w_0	-0.794 ^{+0.243 +0.584} _{-0.244 -0.431}	-0.989 ^{+0.160 +0.413} _{-0.162 -0.291}
w'	-0.446 ^{+0.711 +0.948} _{-0.710 -2.237}	-0.177 ^{+0.571 +0.742} _{-0.574 -1.984}
$\sigma^2(w_0, w')$	-0.192	-0.112
w_0	-0.813 ^{+0.293 +0.704} _{-0.296 -0.508}	-1.017 ^{+0.199 +0.503} _{-0.200 -0.350}
w_a	-0.510 ^{+1.265 +1.792} _{-1.259 -3.620}	-0.039 ^{+1.045 +1.429} _{-1.052 -3.173}
$\sigma^2(w_0, w_a)$	-0.410	-0.245

nificant increase in the number of observed SNe Ia between redshifts of 1 and 2, complemented by BAO and weak lensing cosmography measurements (such as expected from the JEDI mission concept of the Joint Dark Energy Mission), should dramatically tighten dark energy constraints and shed light on the nature of dark energy (Wang 2000a; Wang et al. 2004).⁶

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⁶ We have assumed a flat universe for all our results. When data of significantly larger quantity and higher quality become available from JDEM and the next generation ground-based surveys, it will be possible to constrain the curvature of the universe as well.

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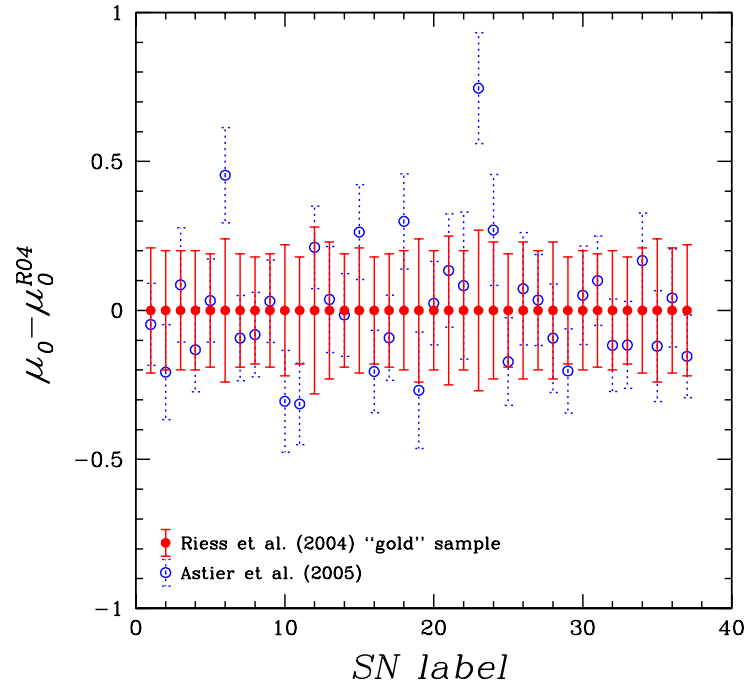


FIG. 1.— The difference in estimated distance moduli for 37 SNe Ia included by both the Riess et al. (2004) "gold" sample and Astier et al. (2005); there is clearly significant scatter due to difference in analysis techniques (Wang 2000b).

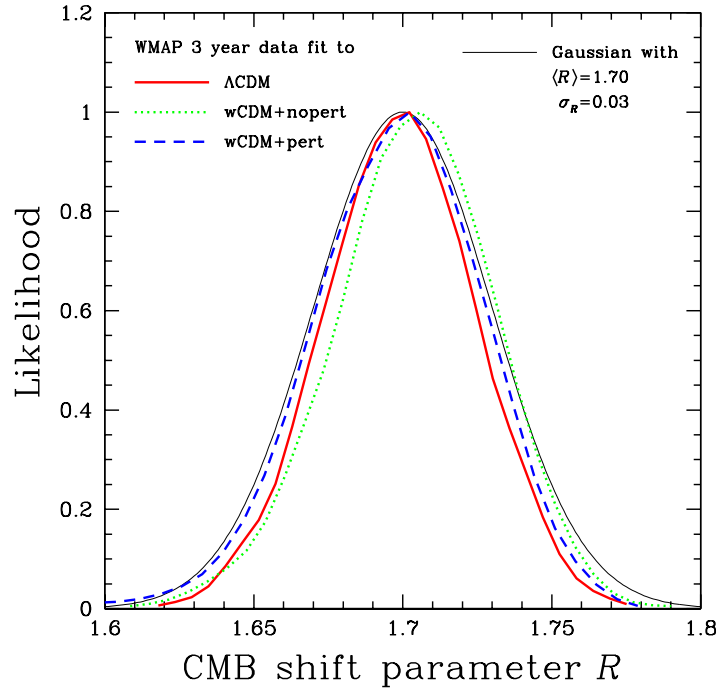


FIG. 2.— The probability distribution of R for three different classes of models.

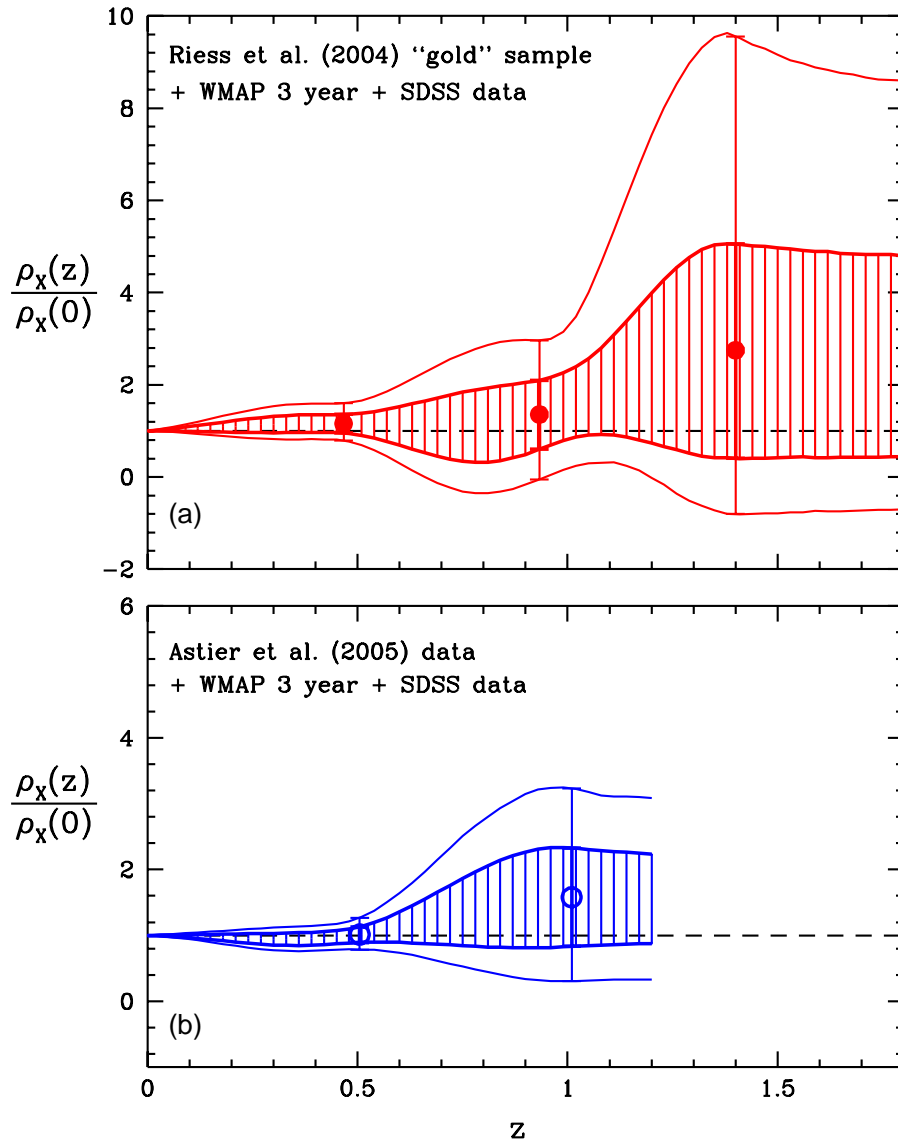


FIG. 3.— Dark energy density $\rho_X(z)$ measured using SN Ia data (Riess et al. 2004; Astier et al. 2005), combined with the WMAP 3 year data, and the SDSS BAO data. The 68% (shaded) and 95% confidence contours are shown. Note that beyond $z_{\text{cut}}=1.4$ (upper panel) and 1.01 (lower panel), $\rho_X(z)$ is parametrized by a power law $(1+z)^\alpha$.

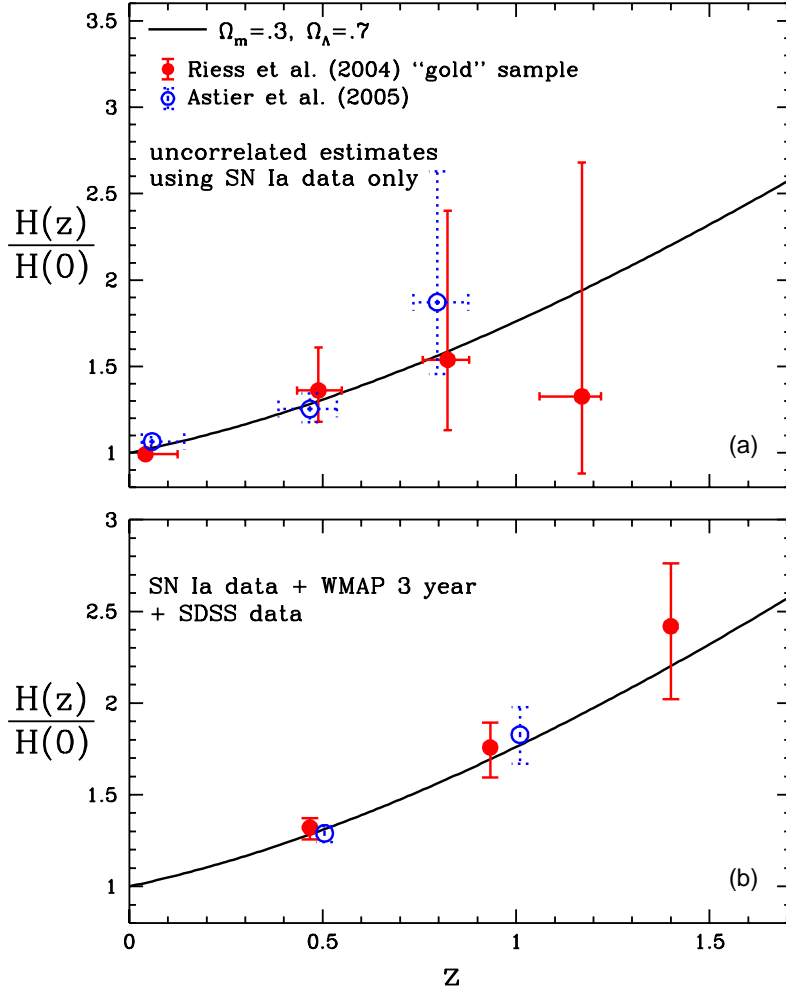


FIG. 4.— Upper panel: the uncorrelated $H(z)$ estimated using only SN Ia data. Lower panel: the $H(z)$ corresponding to the $\rho_X(z_i)$ shown in Fig. 3. The error bars indicate the 68% confidence limits.

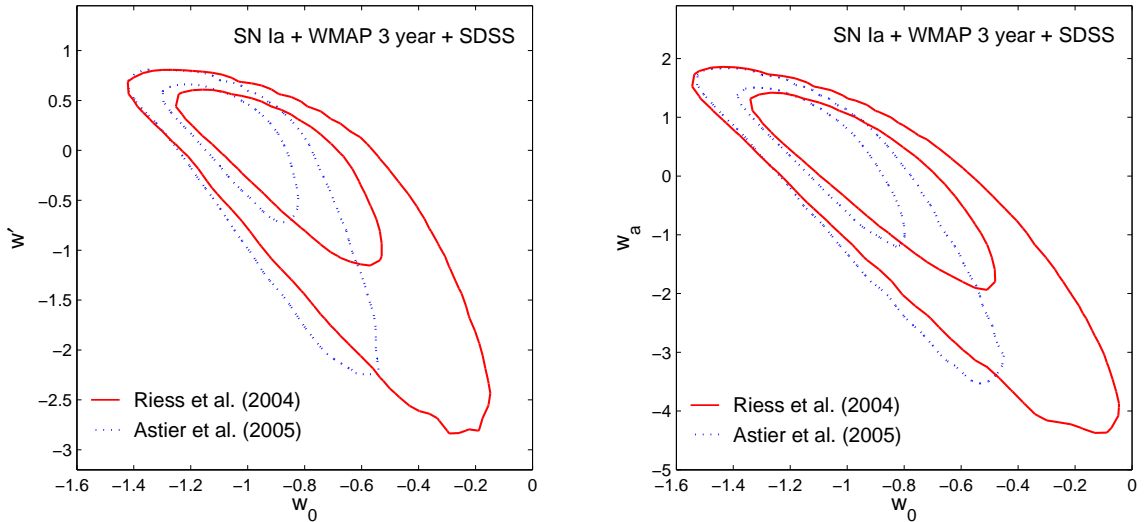


FIG. 5.— The 68% and 95% joint confidence contours for (w_0, w') and (w_0, w_a) .